Small Strain Stiffness Model for Crisp

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Model implemented in CRISP 99 by Amir Rahim
Amended by Amir Rahim in CRISP 2002.4

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Introduction

Recent advances in strain measuring devices for soil samples have shown that soils exhibit very large stiffness at very small strains of the order of 0.004%. Jardine et. al (1984) published laboratory measurements of soil stiffness using local strain measurements devices that could resolve mean axial strains as low as 0.002%. Their data show undrained 'elastic' moduli continually reducing from strains as low as 0.005% until failure is approached.

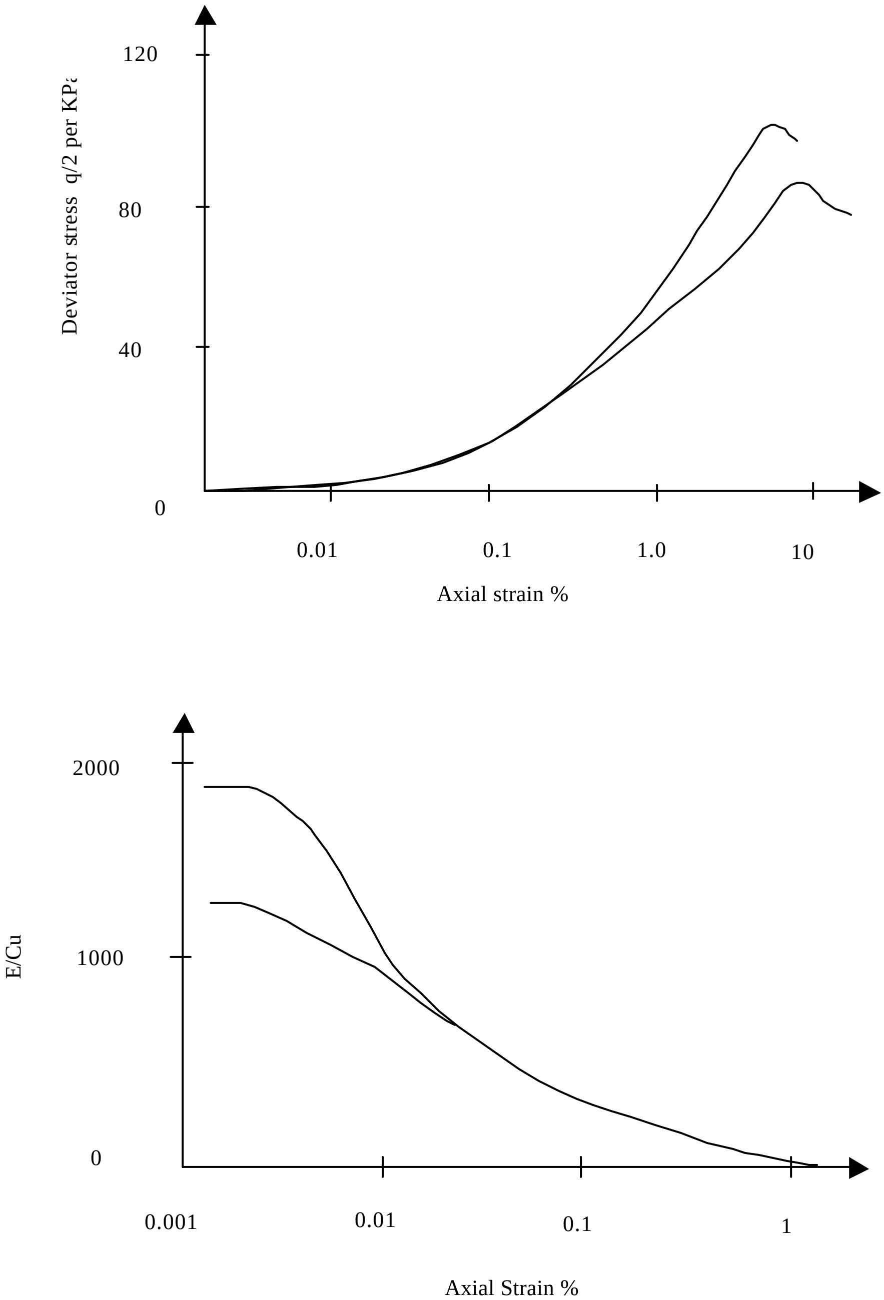


Figure 1 Data for low strain stiffness of London Clay after Jardine et. al. (1984)

Stiffness Formulations

Professor Mike Gunn introduced a simple model based on the laboratory results shown above.

The undrained non-linear 'elastic' response of the soil is given by:

$q=a\varepsilon^{n}$

where q is the deviator stress, ε is the deviator strain and a and n are soil parameters obtained as described below. This power law expression can be manipulated in a very simple way to give expression for the secant stiffness

sec.Eu=$a\varepsilon^{n-1}$

and the tangential stiffness as
tan.Eu=$na\varepsilon^{n}$

The parameters a and n for the model are recovered from the secant Young's modulus measures at two strain levels in an undrained triaxial test:

$$E_{u1}=a\varepsilon_1^{n-1}$$
$$E_{u2}=a\varepsilon_2^{n-1}$$

Using Eu1 and Eu2 together with the equation for the secant stiffness, one can obtain an expression for n as follows:

$$n=1+\log(E_{u1}/E_{u2})/\log(\varepsilon_1/\varepsilon_2)$$

and

$$a=E_{u1}\varepsilon_1^{1-n}$$

For example, if we adopt values of Cu=100 kPA, then
$\sec E_u/C_u=1000$ for a strain of 0.01%
$\sec E_u/C_u=400$ for a strain of 0.1%,
then the above would give values of a=2500kPa and n=0.6.
The model introduced by Mike Gunn incorporates a Tresca yield surface to allow for plastic yielding when the deviator stress reaches the limit given by the shear strength Cu. In addition, the model allows for the variation of C and a with depth according to the formulae:

$$C=C_o+m_c(y_o-y)$$
$$a=a_o+m_a(y_o-y)$$

where Y_o is the elevation at which the undrained Cohesion C_o and the parameter a_o are measured, m_c and m_a represent the rate of change of C and a with depth respectively.

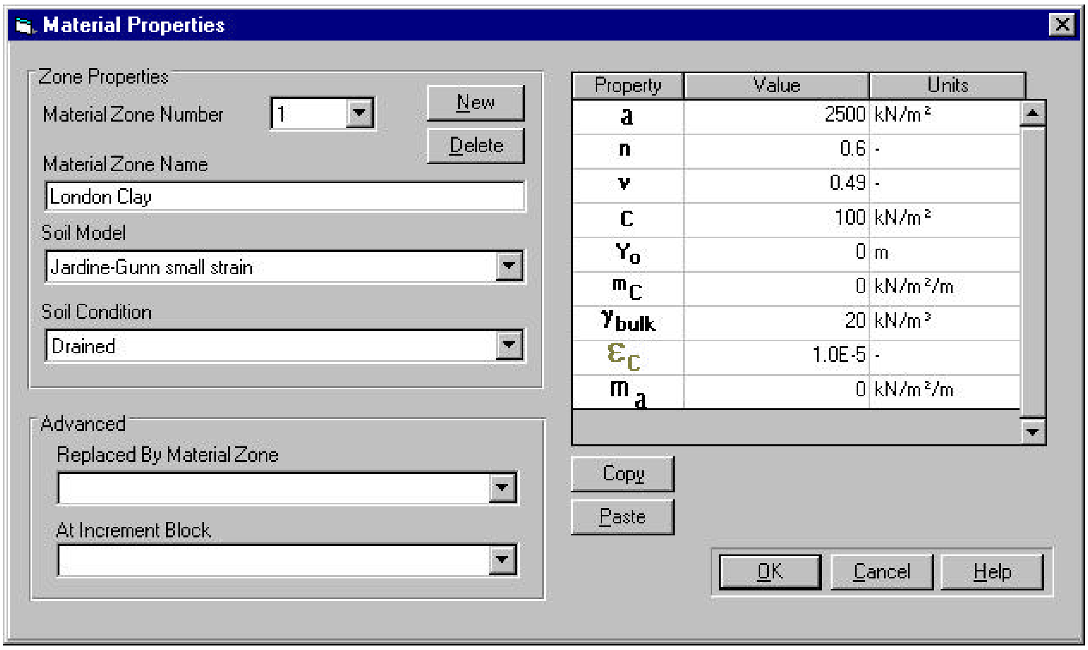


Figure 2 Material Properties in SAGE-CRISP v 4

There is a value of strain (ε_c) below which the stiffness is taken to be constant (and equal to the secant stiffness at ε_c), so the actual stress strain curve and variation of modulus with strain is as shown in figure below, where a value of ε_c=0.001% is assumed.

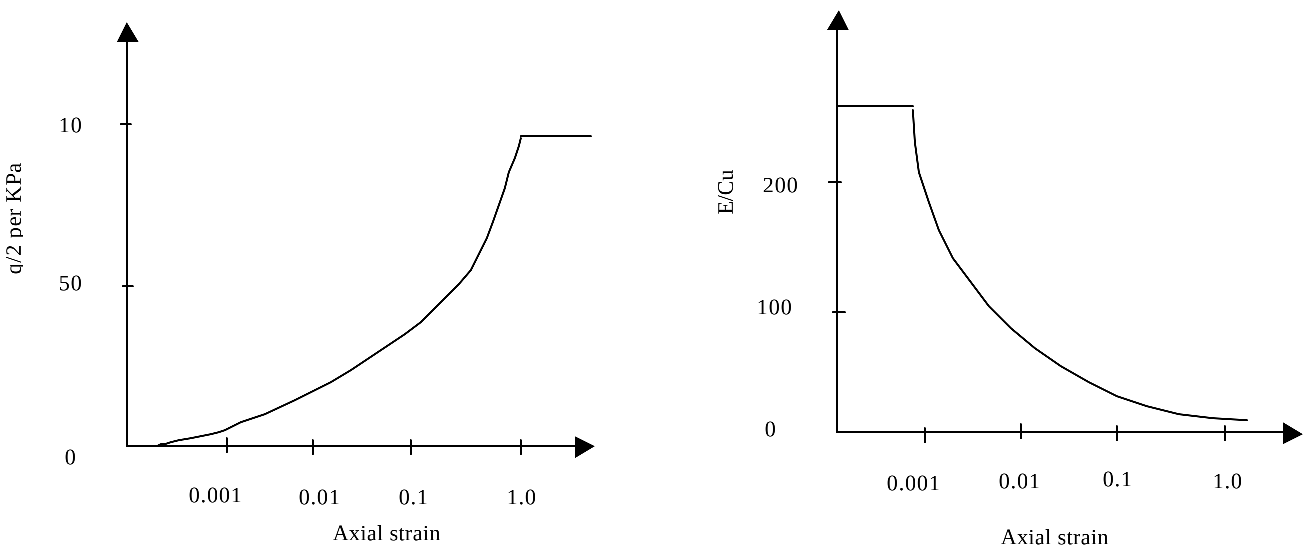
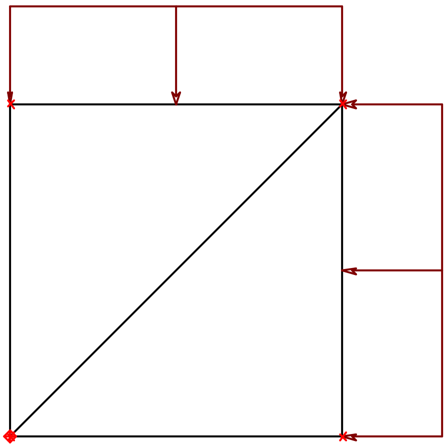


Figure 3 Stiffness relations using q=ae^n for data of Jardine et. al. (1984)

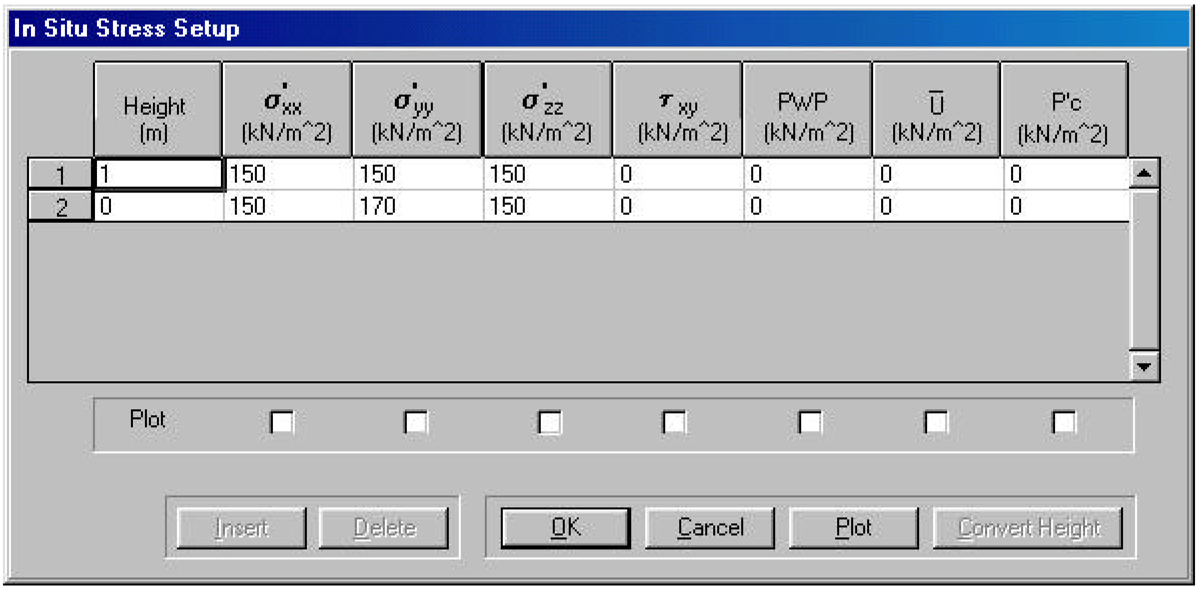
Validation Tests

We setup a simple finite element mesh for a triaxial test. This will consist of two LST axi-symmetric element as shown below.



In order to satisfy equilibrium of stresses, we apply a pressure of 150 KN/M^2 to each side as shown above.

The in-situ stresses are applied as shown below. Notice that σ_y increases with depth due to unit weight of soil



In Situ Stress Setup

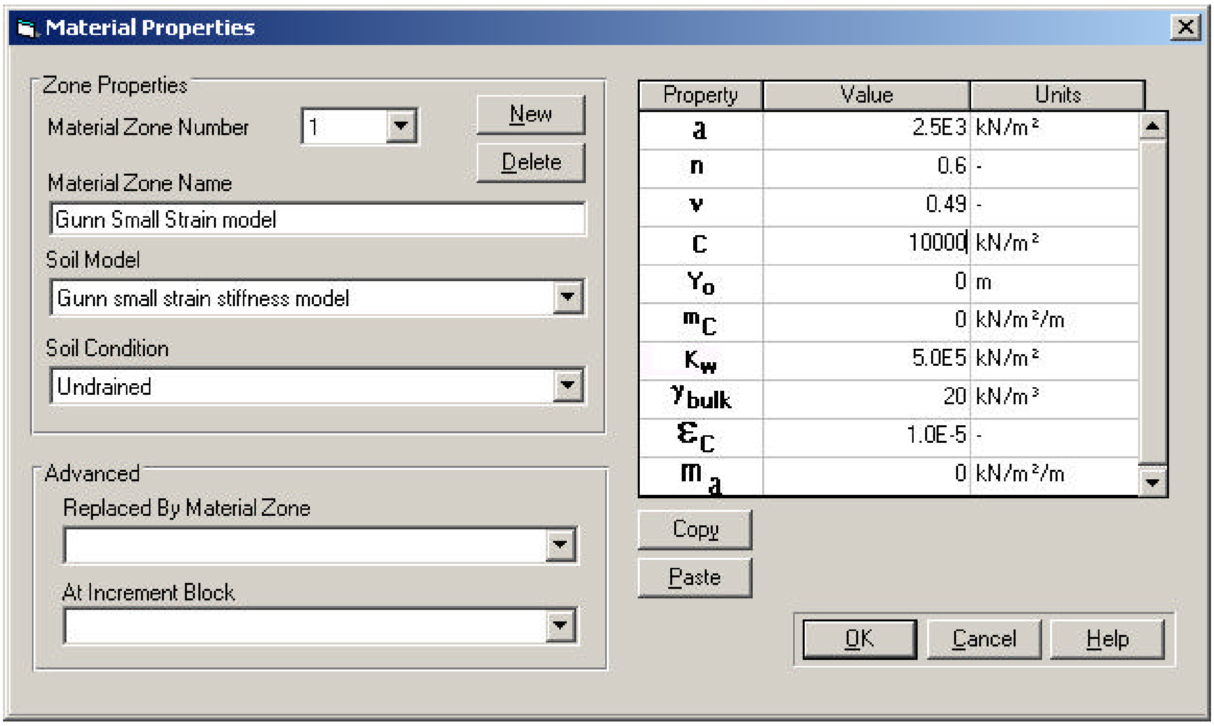
	Height (m)	σ'_{xx} (kN/m^2)	σ'_{yy} (kN/m^2)	σ'_{zz} (kN/m^2)	τ_{xy} (kN/m^2)	PWP (kN/m^2)	$\bar{U}$ (kN/m^2)	P'c (kN/m^2)
1	1	150	150	150	0	0	0	0
2	0	150	170	150	0	0	0	0

Plot

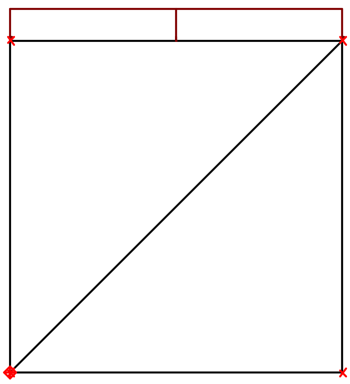
Insert Delete OK Cancel Plot Convert Height

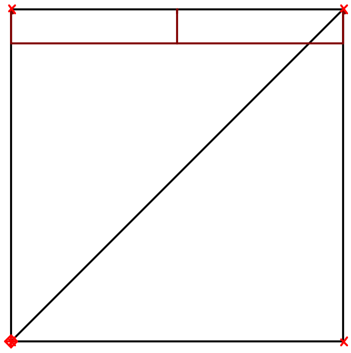
Test 1

For the first test, we will set the shear strength C to a very high value so that the sample behaves elastically.



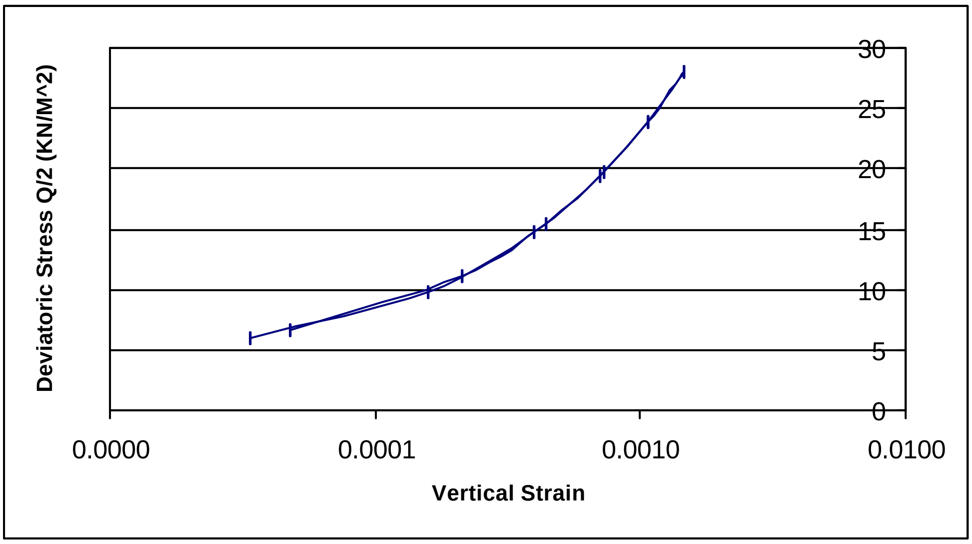
We now apply a vertical pressure of 50 KN/M^2 in one load block and then reveres this pressure in the subsequent load block as shown in the two figures below.





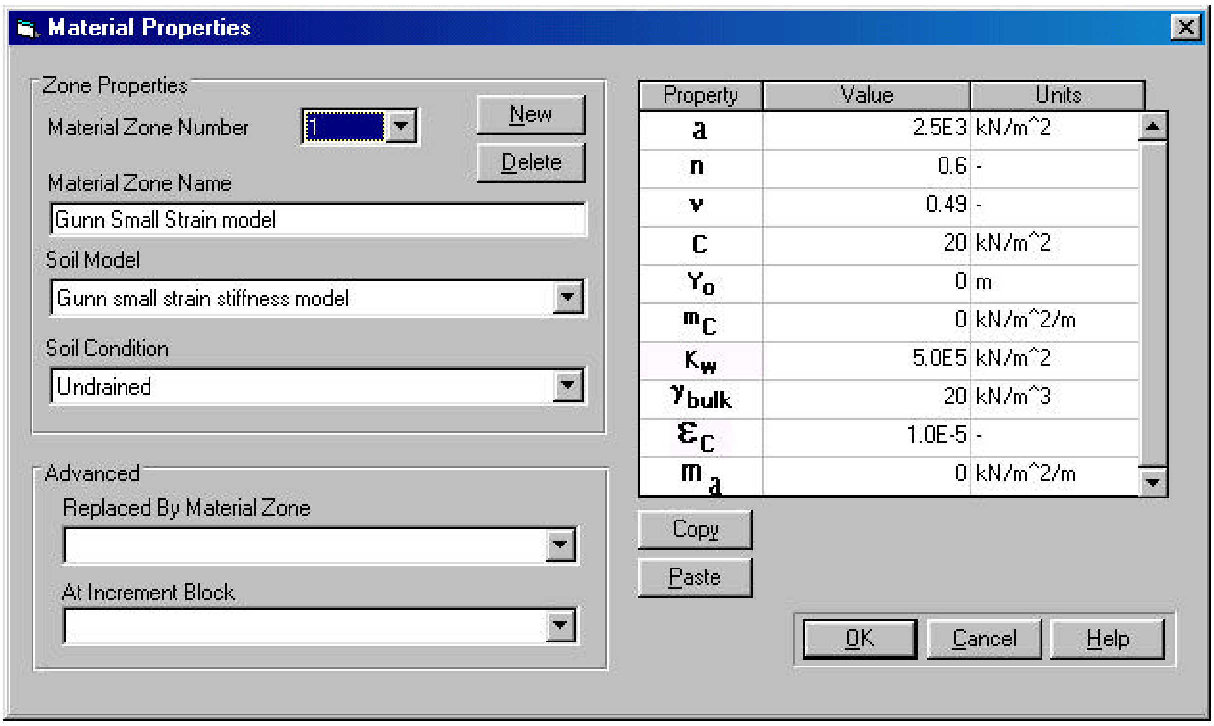
We use 10 increments for each load block. In addition we use the option (apply out of balance forces), or the fully iterative solution using Modified Newton Raphson method. This is done through File>Project Setup.

Plotting the results of vertical strain (natural log) against deviatoric stress shows that the unloading path follows the loading path as shown below

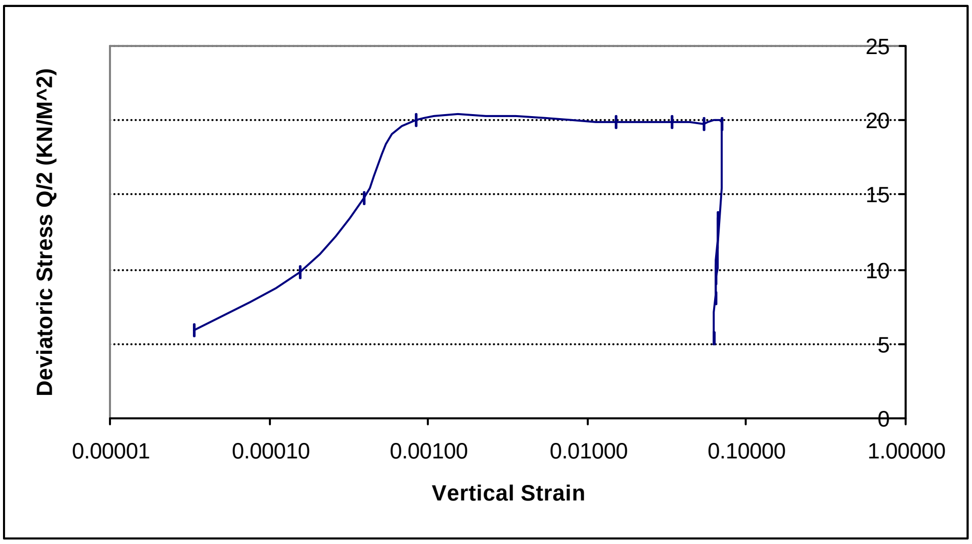


Test 2

We now use a value of 20KN/M^2 for the shear strength as shown below and re-run the analysis above



The graph of q/2 against natural log of vertical strain shows that the graph reaches a limit of 20 which corresponds to the undrained shear strength specified in the material properties.



Concluding remarks

The assumptions made in deriving the model, and the details of its implementation into CRISP mean that there are some important limitations on its use.

- The model has been developed on the basis of data from undrained tests, and so it should be used to predict undrained deformation (or drained with poisson's ratio =0.49)
- The model is designed for modelling situations where loading is monotonic. Contours of constant elastic shear modulus are circles in the π plane of principal stress space, centred upon the point corresponding to the stresses at the start of the analysis. In other words, the stiffness in unloading is just the same as the stiffness in loading at the equivalent strain level.

Before the relationship $q=a\varepsilon^n$ was adopted, previously proposed non-linear elastic relationships for soil due to Naylor (1981) and Duncan and Chang (1969) were considered and rejected. Both of these models have two parameters describing a non-linear stress-strain curve and the parameters can be obtained in a similar fashion to the procedure described above, fitting the non-linear curves at low strain values. If this is done, for example with the data quoted above, both of these models predict that the soil fails at a value of q about one third of the value actually seen. In practice one would use these models with the maximum value of q correctly represented, but this will be at the cost of a poor representation of stiffness at some values of low strains. In contrast, the relationship $q=a\varepsilon^n$ gives a reasonable fit for stiffness until plastic yielding starts (at a strain of about 1.5%)
In fact the model described here has some similarity to that described by Jardine (1986). The main difference seems to be that their equation matches the data more precisely at the cost of some extra complexity in the form of the equation and the derivation of material parameters.

References

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